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ABSTRACT

A series of eight videoconferences was designed to help educational professionals improve schools by applying knowledge gained from research. During the videoconference programs, which are transmitted by satellite to facilities with receiving capacity, viewers can interact by telephone with the program presenters. Each program covers a specific topic, and each has a "content partner" (learning center, institute, teachers' association, or other specialized group) that supplies expertise in the program area. The format of the program involves the presentation of two stories depicting classrooms or communities that have successfully integrated research with practice, and a question-and-answer session with the studio audience and remote viewers. In this document, a review of research on mathematics instruction is followed by an overview of the second program in the series, which covers the topic of mathematics instruction and learning. In this program, the first story illustrates a "cognitively guided instruction" approach that has resulted in high levels of mathematics achievement for students in a first grade class. The second story examines the work of the Algebra Project in Boston and Chicago. In this project, a middle school mathematics curriculum has been designed to address the crisis in mathematics education among inner-city students. A series of follow-up activities relevant to the topic of mathematics instruction is described. Additional materials include biographical information on videoconference participants, a list of resources on mathematics instruction, a glossary of terms, and a reference list of 16 items. (BC)

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SCHOOLS THAT WORK
The Research Advantage

GUIDEBOOK

2

Children as Problem Solvers

PS 020325

An Eight-Program Videoconference

presented by the
North Central Regional Educational Laboratory
and the PBS Elementary/Secondary Service
in partnership with the
National Center for Research in Mathematical Sciences Education
and the National Council of Teachers of Mathematics

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NCREL

The North Central Regional Educational Laboratory (NCREL) is one of ten federally supported regional educational laboratories in the country and serves the states of Illinois, Indiana, Iowa, Michigan, Minnesota, Ohio, and Wisconsin. NCREL works with education professionals in the region to support school restructuring to promote learning for all students, especially those most at risk of academic failure in the most rural and urban settings.

NCREL was created in 1984 by seven chief state school officers and 19 deans of education from colleges and universities. It is governed by a 29-member board of directors composed of the seven chief state school officers and representatives from higher education, business, labor, agriculture, local and intermediate educational agencies, and communities.

The Lab's efforts to respond to regional needs are focused around five major program areas which direct our efforts to those in greatest need and allow us to concentrate our resources in areas likely to result in the greatest impact: curriculum, instruction, and assessment; early childhood and family education; professional development; rural education; and urban education.

As an educational laboratory, NCREL assists schools, families, and communities in identifying and addressing the needs of our children and students and in redesigning schools so that they have a greater capacity to create learning environments appropriate to, and supportive of, the full diversity of student needs and circumstances. We believe a growing body of expertise embodied in theory, research, and practice is pointing toward ways in which the nation can fundamentally rethink, or restructure, its schools.

PBS

The PBS Elementary/Secondary Service (ESS), through and on behalf of PBS member stations, serves the instructional needs of students and teachers nationwide. ESS distributes educational programming and related materials for K-12 classroom use; promotes the use of PBS's primetime programming as a curricular resource; provides professional development for educators; and serves as a national advocate for the use of video and related technologies for learning.

The PBS Adult Learning Service (ALS) offers college-credit television courses through local partnerships of public television stations and colleges. Since 1981, more than 1,500 colleges, in cooperation with 300 public television stations, have enrolled more than one million students in ALS-distributed courses. In August 1988, ALS launched the PBS Adult Learning Satellite Service (ALSS) as a direct satellite service for higher education, offering a wide variety of programming.

National Center for Research in Mathematical Sciences Education

Each program in the videoconference has a content partner, or partners, that have expertise in the area. We have selected the National Center for Research in Mathematical Sciences Education at the University of Wisconsin-Madison as a content partner for the Mathematics Program because it is actively involved in spearheading reform in mathematics education and in developing models of authentic assessment. In the Center are seven working groups which conduct research in algebra, geometry, quantity, number, and statistics. The Center is funded by the Office of Educational Research and Improvement within the United States Department of Education and by the National Science Foundation. The Center has played a major role in the important events and research development presented in this guidebook and videoconference program. Specifically, the Center helped NCREL identify presenters, special guests, and schools, as well as worked with us to develop the guidebook.

National Council of Teachers of Mathematics

In addition to the Center, we have chosen the National Council of Teachers of Mathematics (NCTM) as a content partner for this program. Since 1920, the National Council of Teachers of Mathematics has been dedicated to the improvement of school mathematics instruction at all levels. Through its many publications, conferences, and other services, this professional organization of more than 82,000 members provides a forum for discussing new developments, sharing innovative classroom experiences, and evaluating trends in the teaching of mathematics. NCTM and its more than 230 affiliated groups in the United States and Canada form a network that serves as a clearinghouse of information and resources for all topics related to mathematics education.

What is a Videoconference?

The North Central Regional Educational Laboratory (NCREL) and the PBS Elementary/Secondary Service (ESS) have been producing videoconferences together since 1987. Via satellite, videoconferencing allows the exchange of information across town or across the country. A growing number of organizations use this delivery system for staff development, corporate meetings, training sessions, and entertainment events.

Videoconferencing works like this: An "uplink" station transmits the live program to a communications satellite which then beams it to "downlink" sites. Any facility with a satellite dish and the proper receiving capacity can be a downlink site. When combined with a telephone hookup, videoconferencing becomes "interactive" with one-way video (viewers see the presenters on monitors although presenters cannot see the viewers) and two-way audio (viewers can call in and speak to the presenters). Another way to include an interactive component is to have a viewer call-in period following the live satellite transmission.

In 1990, NCREL and PBS produced a nine-part staff development videoconference series, *Restructuring to Promote Learning in America's Schools*. Although not geared specifically toward staff development, *Schools That Work: The Research Advantage* has been designed to help viewers develop the kinds of world-class schools that ensure a successful future for all of us.

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SECTION 1

Overview of the Videoconference Series

The President of the United States and the National Governors Association recently set forth six goals to be achieved by the year 2000 (see page 3). *Schools That Work: The Research Advantage* is designed to provide educational research for schools and communities as they reflect upon and seek to implement these national education goals. A recent Gallup poll shows that most Americans strongly support the national goals. Each goal was rated as a high priority by at least 84 percent of those polled. However, at least 47 percent felt that reaching these goals by the year 2000 is unlikely.

The North Central Regional Educational Laboratory (NCREL) and the PBS Elementary/Secondary Service (ESS) are committed to restructuring schools to promote learning. We believe that every child can learn. We also believe that it is in our nation's utmost interest to develop world-class schools that achieve these goals, for the sake of our children.

Scattered throughout America are schools that work. These schools involve teachers, students, administrators, parents, community members, and businesses. They illustrate how the application of learning research can lead to improved education and to the achievement of the national goals. These examples give us hope that we all can make the difference. This videoconference is designed to help viewers develop schools that work, schools that aim to meet the national goals using the research advantage.

Topics and Schedule

1991 (4:00-5:00 pm ET)	**1992** (4:00-5:00 pm ET)
Reading—October 10	Early Childhood Education—February 13
Mathematics—October 24	Drug-Free Schools—March 12
Science—November 14	School-to-Work Transition—April 2
Assessment—November 21	Interagency Collaboration—May 7

Audience

Schools That Work: The Research Advantage is designed primarily for teachers, administrators, and curriculum specialists, as well as staff of state education and intermediate service agencies. However, its fast-paced, informative style will appeal to school board members, parents, policymakers, Chapter I staff, teacher educators and their students, test makers, staff developers, and researchers interested in school improvement.

Videoconference Programs

Videoconference 1

Reading ▪ October 10, 1991 ▪ Children as Strategic Readers

Content Partner: Center for the Study of Reading (CSR), University of Illinois, Urbana-Champaign

Videoconference 2

Mathematics ▪ October 24, 1991 ▪ Children as Problem Solvers

Content Partners: National Center for Research in Mathematical Sciences Education, University of Wisconsin, Madison, and the National Council of Teachers of Mathematics

Videoconference 3

Science ▪ November 14, 1991 ▪ Children as Explorers

Content Partners: Institute for Research on Teaching, Michigan State University, and Project 2061, American Association for the Advancement of Science (AAAS)

Videoconference 4

Assessment ▪ November 21, 1991 ▪ Alternatives for Measuring Performance

Content Partner: National Center for Research on Evaluation, Standards and Student Testing (CRESST), UCLA

Videoconference 5

Early Childhood Education ▪ February 13, 1992 ▪ Meeting Children's Needs

Content Partners: National Association for the Education of Young Children (NAEYC) and the National Association of Early Childhood Specialists in State Departments of Education

Videoconference 6

Drug-Free Schools ▪ March 12, 1992 ▪ Preparing Students for Drug-Free Lifestyles

Content Partner: Midwest Regional Center for Drug-Free Schools and Communities

Videoconference 7

School-to-Work Transition ▪ April 2, 1992 ▪ Preparing Students for Work in the 21st Century

Content Partners: Institute on Education and the Economy, Teachers College, Columbia University, and the U.S. Chamber of Commerce

Videoconference 8

Interagency Collaboration ▪ May 7, 1992 ▪ Integrating Community Services

Content Partners: Study Group on School-Linked Integrated Services sponsored by the U.S. Department of Education and the Department of Health and Human Services

Videoconference Objectives

- To provide examples of how research empowers teachers, students, and whole communities
- To promote meaningful and self-directed learning for all students
- To focus national attention on issues of equity and excellence as schools and communities implement the national goals
- To demonstrate alternative ways that educators, parents, policymakers, and community members can work together to redesign schools for America's children
- To provide high-quality programming with up-to-date research for professional development libraries for educators
- To encourage schools and communities to reflect on the national education goals, well informed by the best of research and practice

The National Education Goals

Goal 1: By the year 2000, all children in America will start school ready to learn.

Goal 2: By the year 2000, the high school graduation rate will increase to at least 90 percent.

Goal 3: By the year 2000, American students will leave grades four, eight, and twelve having demonstrated competency in challenging subject matter including English, mathematics, science, history, and geography; and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning, and productive employment in our modern economy.

Goal 4: By the year 2000, U.S. students will be first in the world in science and mathematics achievement.

Goal 5: By the year 2000, every adult American will be literate and will possess the knowledge and skills necessary to compete in a global economy and exercise the rights and responsibilities of citizenship.

Goal 6: By the year 2000, every school in America will be free of drugs and violence and will offer a disciplined environment conducive to learning.

Rationale for the Videoconference Program Topics

The topics for this series were selected in part because they represent major areas of educational research and in part because they are all relevant to the national education goals. Although each topic area in the series involves different departments and school staff, it is important to understand that schools ultimately are part of one system. What happens in one area of schooling influences what happens in another area. This videoconference presents two sets of thematically related programs.

- The theme connecting the first four programs (Reading, Mathematics, Science, Assessment) is that research in these areas forms the vision of learning and teaching that drives curriculum, instruction, and assessment. Further, many common threads exist in each research base such as teaching for understanding and collaborative learning.
- The theme connecting the last four programs (Early Childhood Education, Drug Education, School-to-Work Transition, Interagency Collaboration) is that traditionally these topics have been considered somehow outside the core of schooling—both conceptually and politically. Our intent is to demonstrate that they are vital for the achievement of all children. Further, the collaborative learning and higher order thinking, together with their assessment, needed in the subject disciplines are also vital to early childhood education, preparing students for drug-free schools, and the work environment.

Accordingly, NCREL and PBS strongly recommend that schools and community members watch all eight programs to develop a common language and common objectives. Indeed, we hope that schools and community members will use this series as a vehicle for debate and reflection about school reform and implementing the national goals.

"We feel that we're building the base to make a breakthrough with the Algebra Project. But it's a question of how to make teachers feel comfortable enough to be freed up to be innovative.

—Doug Gills

Format of Each Videoconference Program

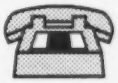
Live Satellite Transmission (1 hour)

For educational research to have a positive impact on schooling, it must be supported by effective classroom practices. With this in mind, the satellite transmission portion of each videoconference will focus on two "stories" depicting actual classrooms or communities that have successfully integrated research into practice. Each story will be followed by a question-and-answer session with the live studio audience. Remote viewers will be able to call in at any time during the program and give their questions to the telephone operator. The moderator will ask selected questions to the presenters throughout the program. Questions not answered on the air may be answered by phone or in print.

The first story will demonstrate effective classroom practices. The second will portray the rich teaching and learning environment created through community involvement in schooling. Each program in the videoconference series draws upon the best of research and practice to address critical questions that schools and communities need to resolve if they are to implement the national goals. Specifically, each program will illustrate the following:

1. How research has changed an individual teacher and the lives of his/her students
2. How schools have used research in the community by involving parents or community members in planning, instruction, or assessment or have enabled teachers and students to engage in learning outside the traditional boundaries of the school

Participant Call-In (30 minutes)



Immediately following the satellite transmission of each videoconference, participants will be given the opportunity to "call in" with comments and questions for videoconference presenters. This segment does not include live video transmission. However, callers will speak directly to the presenters. Because this segment provides one-to-one contact between presenters and participants, it is the key to the interactive nature of the conferences.

SECTION 2

What Does Research Say About Mathematics?

Goal 4 of the national education goals explicitly calls for improvement in instruction resulting in students who are first in the world in mathematics achievement. Research in mathematics learning and instruction, when coupled with the Curriculum and Evaluation Standards for School Mathematics produced by the National Council of Teachers of Mathematics, provides a powerful mechanism for achieving this important national goal. In the following sections we present:

- An historical context of mathematics education in the United States
- Major shifts in mathematics curricula
- Important findings about mathematics and learning from cognitive science
- Principles related to mathematics understanding
- Important milestones in mathematics research
- Motivation for change in mathematics education
- Conditions necessary for change in mathematics education

Historical Context

Math curricula in the United States have encountered three major shifts since the turn of the century. Until the early 1960s behaviorism was the predominant view of learning in all subject areas. Behavioristic researchers proposed that when learning occurred, "bonds" were formed between a stimulus in the environment and a response, or behavior, by the learner. To learn a simple addition fact, a bond was formed between an environmental stimulus (for example, "4 + 4") and the correct learner response ("8"). Even though many classroom teachers did not fully implement the principles of behaviorism, drill and practice on math facts and computation, thought to "strengthen" the bonds between questions and responses, became the chief mode of instruction.

Soon after 1957, the first curricular shift occurred. The launch of Sputnik by the Soviets shocked America and made clear that (1) we had fallen behind in mathematics and science achievement and that (2) mathematics and science curricula in our elementary and high schools were to blame. Thus, many of the nation's top mathematicians and scientists were assembled to develop new math and science programs, primarily intended for high school students whose career path included engineering and other science-related occupations. One of the results of this effort was "new math" which featured a highly abstract and conceptual focus. America did not fully embrace this "new math." Consequently, it's impact on improving mathematics achievement was perceived as less than expected. A cry to return to the "3 Rs" rang out. In this "back to basics" curricular shift, mathematics instruction returned to a drill and practice orientation and remained so until the end of the 1970s.

In the late 1970s, cognitive science—the study of the mind—overtook the longstanding behavioral approach. As a result, the view of math changed from "numbers and computations" to "math as problem solving," representing the third curricular shift. Educational psychology began to focus on learning and problem solving in specific content areas rather than attempting to discover content-independent principles of learning. In studies of mathematical problem solving, an emphasis is placed on (1) uncovering mental processes involved in expert performance within a content domain, (2) analyzing novice performance, and (3) characterizing the mental changes involved when a novice develops into an expert. A major thrust of this research has concentrated on the role of (1) children's prior knowledge, (2) out-of-school experience, (3) informal mathematics knowledge, and (4) the knowledge and beliefs of the teacher. Research about children's development rounded out this research agenda. Over the last decade, knowledge in these areas has grown considerably. Consequently, researchers and practitioners have a much better grasp of what it means for a person to learn and understand math. This has led to the development of more effective instructional programs.

Major Shifts in Math Curricula

	Drill and Practice View	Problem Solving View
Research Base	Behaviorism	Cognitive science
Goals of Math Instruction	Mastery of isolated facts and skills	Understanding of mathematical concepts to apply to new problem situations
Goals of Learning	Applying computation skills and memorizing math facts to get the right answer	Problem solving, understanding the conceptual nature of the problem, knowing when to employ skills and facts as tools to solve the problem
View of Learner	Passive; vessel receiving knowledge from external sources	Active; connecting intuitive and prior knowledge to new formal knowledge

Important Findings from Cognitive Research

One of the most important contributions that cognitively based research in mathematics has provided is an explanation about the nature and importance of understanding.

1. Understanding can be thought of as an active process of forming—or constructing—connections between knowledge. To understand means to have grasped the relationships among mathematical facts, procedures, concepts, and principles. It also means forging relationships between new knowledge and what one already knows, both from in- and out-of-school experiences.
2. Research is beginning to show how students can construct the most meaningful relationships between mathematical knowledge. This has direct implications for instruction, particularly the importance of teaching for mathematical understanding rather than skill development.
3. It is important to teach for understanding in math because:
   - Understanding is necessary in order to retain knowledge and apply it to new situations.
   - Research shows that teaching for understanding does not lead to a decline in skills.
   - Skilled performance depends upon understanding.

"To me the Algebra Project is just a tremendous opportunity to change a situation and not simply sit back and say 'nothing can be done' because something can be done."

—Jackie Rivers

Principles Related to Mathematics Understanding

Principle	Implications
Thinking is a continual process and is fundamental to learning mathematics; the development of mathematical thinking does not depend on the acquisition of basic skills.	Instruction should provide opportunities for thinking and reasoning throughout the math curriculum; basic skills instruction should not be separated from thinking skills instruction; if students practice skills before they understand the underlying concepts, they will have difficulty learning the conceptual knowledge later, and lack the power to use mathematics.
Thinking and knowledge are not separate components of mathematical competency, but rather are highly related; the quality of mathematical thinking and reasoning is dependent on what one knows, and the development of what one knows is dependent on mathematical thinking.	Students must be provided opportunities to construct mathematical knowledge through thinking and reasoning, and to think and reason with their mathematical knowledge, as they attempt to make sense of their experiences through real world problem solving.
Thinking and motivation are closely related; successful problem solvers have dispositions to question, elaborate, justify, and regulate their learning.	Students must be allowed to participate in environments where these dispositions are valued, where students can practice them, and where students are accountable for their learning.
Students actively construct knowledge through doing and problem solving rather than by passively assimilating information from the teacher and textbook.	Instruction should help children understand mathematical concepts rather than just give them isolated facts and procedures; students should solve problems in order to construct formal mathematics knowledge (e.g. representing physical events with algebraic symbolism).
Students acquire informal systems of mathematical knowlege (for example, counting on fingers) outside of school which forms the basis for formal mathematical understanding.	Instruction should be designed to build upon children's informal knowledge by linking in-school, formal mathematics concepts (e.g. variables and equations) to real world problems and contexts.
Problems are solved in various ways and knowledge is often constructed by groups of people.	Students should be encouraged to solve mathematical problems collaboratively, listen to and challenge each other, and value different problem solving approaches.
The use of calculators and computers does not lead to a decrease in computation skills.	Allow students to use these tools making sure that they understand why, how, and when to use them.
Mathematical knowledge consists of multiply-linked representations (physical event, pictorial representation, informal language, structured language, symbolic representation).	Instruction should help students progress from everyday, common events (concrete) to symbolic representation (abstract).

Important Milestones in Mathematics Research Instruction

1. There has been a shift in emphasis toward developing teacher beliefs about learning mathematical and content knowledge because canned instructional programs often fail if teachers do not adopt the beliefs, purposes, and value of the materials.
2. Teaching for understanding is now informed by research that has provided maps of how children's knowledge changes as they move from informal mathematical knowledge to formal.
3. Teaching is coming to be viewed as a problem-solving, decision-making process rather than as the imparting of knowledge. Thus a new emphasis on continuous (ongoing) assessment in the classroom is occurring as teachers make moment-by-moment instructional decisions.
4. In 1989, the National Council of Teachers of Mathematics developed and published *Curriculum and Evaluation Standards for School Mathematics,* a set of guidelines designed to "establish a broad framework to guide reform in school mathematics in the next decade" (p. v). This was followed in 1991 by the release of the *Professional Standards for Teaching Mathematics* which provides guidance to those involved in changing mathematics teaching.

We are talking about building the whole child through the Algebra Project process."

Cleeta Ryals

Motivation for Change

Around the nation are examples of excellence in mathematics instruction in which research is playing a key role. These classrooms are encouraging. But it should be made clear that change takes time, effort, and resources. In order to justify the costs associated with improving mathematics instruction, benefits must be carefully weighed. We are convinced, as are the teachers and principals associated with excellence in mathematics instruction, that the benefits clearly outweigh the costs. The motivation for change exists because:

1. Mathematics literacy is one of the most important keys to quality life now and in the future, and for ensuring access and equity for all students.
2. We must generate and nurture student support and provide challenging and meaningful tasks appropriate for the 21st century because all children can learn.
3. Changes in the global economy and in technology require mathematical literacy. Without excellence, America's economic productivity will continue to fall.
4. According to the National Assessment of Educational Progress (NAEP), American students perform adequately on low-level cognitive skills such as computation but in general have trouble with higher level cognitive skills involving understanding and problem solving.
5. According to NAEP, American students perform more poorly when compared with students from other countries.

Conditions Necessary for Change

Experts agree that meaningful change must be systemic. In other words, change must occur in all aspects and levels of the educational system. This system extends far beyond the walls of the mathematics classroom, into the community, nation, and global society. The term systemic also suggests that change in one aspect of the educational system will affect other aspects as well. To complicate matters, each of these aspects also may be in the process of change. Thus in the change process, all those involved must continually communicate and take note of the whole educational system, evaluating current plans within the context of the changing whole. We have identified with our content partners several important conditions necessary for systemic change in mathematics instruction.

1. We must focus on how children learn and understand mathematics and base instructional decisions on this knowledge. This condition calls for a redesign of traditional curriculum, instructional processes, and assessment methods.
2. We must spend time and effort providing teachers with opportunities for professional development. Teachers' knowledge and beliefs about mathematics, the goals of instruction, students' learning, and the nature of instruction are critical and must be addressed as a crucial part of the reform effort.
3. We must involve students, teachers, schools, communities, states, national organizations, and others who have a stake in the mathematics education of our youth in the reform process.

SECTION 3

Overview of Program 2: Children as Problem Solvers

Program Objectives

- To provide the "big ideas" from research and practice in math
- To provide examples of excellence at both classroom and school levels
- To raise the kinds of questions that might be discussed in school and community forums

Program Outline

- The program opens with scenes from classroom demonstrations that serve as a preview of key ideas.
- Moderator Judson Hixson, and Beau Fly Jones, Project Director for the Video Series, both from the North Central Regional Educational Laboratory, introduce the videoconference series and Program 2.
- The first story illustrates how research, specifically Cognitively Guided Instruction, has enabled Sue Gehn's 1st grade class to achieve high levels of mathematics problem solving.
- Researchers Tom Carpenter and Mary Lindquist discuss the "big ideas" from research reflected in Sue Gehn's classroom approach.
- Hixson moderates questions and comments from the audience.
- The second story about the Algebra Project, both in Boston and in Chicago, demonstrates how the community is actively involved.
- Algebra Project associates Jackie Rivers and Dorothy Strong discuss how the Algebra Project reflects the big ideas in math research and issues of equity.
- Hixson moderates questions and comments from the audience.
- Jones asks selected studio audience members to describe how the stories and discussion in this videoconference may influence their thinking.

Overarching Message

1. Developing an understanding of mathematics concepts through problem solving is fundamentally different than knowing math facts and employing rote computational skills.
2. Although programs are available to help schools provide instruction for mathematical understanding, most schools still take a skills-based approach.
3. All children can be taught to be mathematical problem solvers and to understand mathematics concepts.

"Cognitively Guided Instruction has affected my whole array of teaching because in everything I am no longer looking for right or wrong answers, but I am really listening to what the kids are thinking."

—Sue Gehn

About Story 1: Sue Gehn and Cognitively Guided Instruction (CGI)

Sue Gehn teaches 18 students (12 of whom are considered students at risk) in a self-contained 1st grade class in Cottage Grove, just outside of Madison, Wisconsin. While the focus of this videoconference program is on how she achieves excellence in mathematics, Ms. Gehn applies the same concepts of the CGI approach to her other subjects as well. The basis of this approach is that teachers must assess the present knowledge that students have in order to make instructional decisions. As a result, a typical sequence of events that her students experience throughout the day are: (1) Ms. Gehn presents a real life math problem (relevant to her students) to the whole class; (2) students use a variety of ways to represent the problem; (3) students devise and invent ways to solve the problem while Ms. Gehn visits each student, assessing their knowledge through questioning and offers guidance and praise; and (4) children share their representations and problem-solving approaches to the whole class. An entire hour of math might be filled with just three problems to ensure that children develop a solid understanding of the math concepts underlying each. Success of the CGI approach can be measured by the difficulty of problems that her students can solve. In this program you will see 1st graders, who have been in school for only four weeks, solving problems as complex as "How much money will we make from recycling eight pop cans if we get four cents for each can?" Ms. Gehn's personal growth through CGI has been documented in a longitudinal case study. (For more information about CGI, see Section 6: Important Resources in Mathematics.)

Some of the most important **strategies** that she uses in her class include:

- Developing math problems from the everyday experiences of her students; for example, recycling pop cans, spending lunch money, and dividing a treat among class members
- Building on the knowledge that each student has in mathematics and posing questions to extend that knowledge
- Allowing her students to represent mathematical problems with a variety of manipulatives, pictures, and marking systems (chalk, paper)
- Encouraging students to invent new ways to represent and solve math problems
- Probing students to talk about how they understand the math problems and how they solve them
- Having students show the whole class the processes they used to solve the problems
- Requesting that parents ask their children to talk about how they represent and solve the math problems in their homework instead of checking worksheets for correct answers

Indicators that Ms. Gehn uses to assess how well she and her students are doing include:

- Performance-based assessment in which she observes how each student solves math problems
- Feedback from 2nd grade teachers who comment on how well her "last year's students" understand math
- Comments from her students about how they like math and how much they are learning
- Comments from parents about how much their children understand math
- Reflection about what her students could do before and after her involvement with CGI
- Achievement/attitude tests (by CGI researchers) placing her students at or above non-CGI classrooms

Cottage Grove Elementary, Cottage Grove, Wisconsin, is a small town outside of Madison. The school's philosophical view is that all children can learn given the appropriate amount of time and adequate learning conditions. They believe that both cognitive and affective education are equally important. A school-wide support system encourages teachers and students to take risks and try new things. This support system is made up of teachers who are "experts" in a given topic; for example, math, science, or cooperative learning. The "expert" acts as a mentor who provides guidance and support to the other teachers.

Recommended Reading

Carpenter, T.P., & Fennema, E. (in press). Cognitively guided instruction: Building on the knowledge of students and teachers. In W. Secada (Ed.), The reform of school of mathematics in the United States. Special Issue of The International Journal of Educational Research.

Fennema, E., Carpenter, T.P., & Peterson, P.L. (1989). Learning mathematics with understanding: Cognitively guided instruction. Advances in Research on Teaching, 1, 195-221.

About Story 2: The Algebra Project

The Algebra Project is a group representing two generations of black activists who came together to address the crisis in math education among inner-city students. The Project focuses on middle school mathematics and has developed a Transition Curriculum for 6th grade. The curriculum was designed to engage students in the construction of mathematical concepts through a five-step curricular process. This curriculum was developed to bridge the conceptual gap many students experience between arithmetic and algebra, and to prepare students to enter college preparatory math in high school. This process first identifies an event from their own lives, usually a physical event from which the students make pictorial representations or models; then the students discuss and write about the event in informal, intuitive language; next they discuss and write about the event in structured English; and finally, they develop and use symbolic representations of the event. The Project has begun to develop materials for 7th and 8th grades to be used in conjunction with an Algebra I textbook. In this segment you will see two stories: a class and a teacher institute.

A Classroom in Boston

The class you will see, taught by Jane King, was held as part of a summer camp sponsored by Freedom House and the Algebra in Middle Schools Project of the Boston Public Schools. The children participated in the class to get a jump start on the upcoming year. The class illustrates the approach of the Algebra Project. In this approach, teachers do not simply give answers, they act as coaches and guides, constructing experiences that will help students find their own answers by asking increasingly sophisticated questions. Manipulatives are used to help students develop a concrete understanding of algebraic concepts. Students work together to develop, discuss, and refine ideas and conjectures, and solve problems. The goal is to construct mathematical knowledge rather than absorb mathematical information from teachers and textbooks.

Community Action in Chicago

The second aspect of the Algebra Project highlights the involvement of community members which resulted in the Algebra Project being implemented in Chicago. The opportunity for inner-city children to succeed in algebra through the Algebra Project was made possible by proactive community members and staff from Chicago Public Schools who were determined to make a difference. In this segment, you will hear key community members in Chicago trace the history of the project and describe how it is making a difference in the classroom, community, and larger educational system. As a backdrop to commentary by members of these communities, you will see portions of a training institute that 6th and 7th grade teachers received. In this institute, held in Chicago, teachers experience the same progression from physical event to symbolic representation that their students will undergo in the upcoming year. The institute begins with a train ride on the CTA elevated train (the "El") that serves as a common physical event on which to build algebraic understandings. Throughout the institute, new algebraic concepts are built on this common experience. Algebra Project teachers learn not only the concepts to be taught and sound methods of teaching them, but also how to develop a classroom atmosphere of respect.

Recommended Reading

Kamii, M. (1990). Opening the algebra gate: Removing obstacles to success in college preparatory mathematics. Journal of Negro Education, 59, 392-406. [Washington D.C.: Howard University Press].

Klonsky, M. (1991, May). Math reform = changing habits. Catalyst, 5.

Marquand, R. (1989, May 17). Up the opportunity ladder with algebra. The Christian Science Monitor.

Moses, R.P., Camii, M., Swap, S.M., & Howard, J. (1989, Fall). The Algebra Project: Organizing in the Spirit of Ella. Harvard Education Review, 59, 423-443.

Thomas, B. R. (1991, October). Algebra begins to sink in. Catalyst, 9, 15.

Silva, C., & Moses, R. P. (1990). The Algebra Project: Making middle school mathematics work. Journal of Negro Education, 59, 375-391 [Washington D.C.: Howard University Press]. .

Of the programs I know, the Algebra Project is among the most compelling and among the most difficult to pull off well. But it can make a qualitative difference in the lives of kids."

—Charles Payne

Teaching Approaches

Teaching Approach:	Traditional Approach	Ms. Gehn's Approach	Algebra Project Approach
Collaboration	Students often work individually and compete	Children solve problems both as a group and as individuals within a group; students actively listen to each other	Students in groups learn how to listen to each other, discuss and conjecture about mathematical concepts, defend their conjectures, and explain what they mean
Grouping	When grouping occurs, students are placed in homogeneous groups	Students are grouped according to strategies they use or number sizes they can work with; individual and cooperative work in small/large groups	Students are grouped heterogeneously; groups often have a peer "expert" who asks questions and advises; no more than five per group
Activities	Individual seatwork; worksheets; drill and practice; problems from textbooks	Children solve math problems relevent to their daily lives; children use a variety of manipulatives and methods to represent problems	For each concept children experience a physical event, draw pictures of it, talk about it in their language and in structured language, and represent it symbolically
Interaction	Teacher assigns problems from book; teacher demonstrates how to get the correct answer; students work through similar practice exercises	Teacher presents relevant problems; teacher circulates to assess each child's understanding and progress to support their problem solving; students interact with each other as they solve problems	Teacher provides physical event and poses problem; students interact with each other to solve the problem; groups share their approaches with class; teacher circulates and comments/questions groups; parents involved in/out of school
Assessment	Answers on individual worksheets, quizzes, book exercises, and tests graded for correct answers	Assessment is continuous during class and used by the teacher to make instructional decisions; formal assessment (performance-based, interviews) and informal (observation, discussion) used in grading and instruction	Teachers use portfolios, assess comprehension at various levels; use multiple means of assessing progress
Self-Directed Learning	Individual work; finish problems on worksheet independently	Children are encouraged to generate their own unique solutions to problems both individually and in groups	Students learn to be responsible for working toward consensus in group discussions
Skill Instruction	Computation and math facts taught independent of problem solving	Children learn math facts and develop computation skills through problem solving; children invent their own methods of remembering facts and computing	Focus on math concepts; create opportunities to use computational skills with enhanced understanding

Viewers' Guide for Program 2: Children as Problem Solvers

Things to look for during the program...	Spaces for taking notes
How do the teachers and researchers define understanding and successful problem solving? How do their definitions compare to my school?	
How do Sue Gehn and the Algebra Project provide their students with motivation and cpportunities to develop mathematical understanding?	
How is collaboration implemented in each of the two stories?	
What instructional and assessment strategies do the two teachers have in common?	
How are the community members of Chicago involved in curriculum redesign?	
How do the teachers in this program assess excellence in mathematics instruction? In what ways do they involve students in this process?	
How do the approaches presented in this program address the national goals concerning mathematics achievement?	

SECTION 4

Follow-Up Activities for Participants

Activities for Teachers

The examples of excellence in this program clearly show that in world-class schools, teaching is a multidimensional activity. One of the most powerful of these dimensions is that of "teacher as researcher." Not only do teachers need to use research in their practice, they need to participate in "action" research in which they are always engaging in investigation and striving for improved learning. The key to action research is to pose a question or goal, and then design actions and evaluate progress in a systematic, cyclical fashion as the means are carried out. Below are four major ways that you can become involved as an action researcher.

1. Use the checklist found at the end of this section to evaluate your school and teaching approaches.

2. Implement the models of excellence presented in this program. Ask yourself:
   - What outcomes do the teachers in this program accomplish that I want my students to achieve?
   - How can I find out more about their classrooms?
   - Which ideas can I most easily implement in my classroom?
   - What will I need from my school and community?
   - How can I evaluate progress?

3. Form a team and initiate a research project. A research project can be designed to generate working solutions to a problem. The issues for your research group to address are:
   - What is the problem or question we wish to solve?
   - What will be our approach?
   - How will we assess the effectiveness of our approach?
   - What is the time frame for working on this project?
   - What resources do we have available?
   - What outcomes do we expect to achieve?

4. Investigate community needs and integrate solutions within your class activities. Relevant questions include:
   - What needs does the community have in terms of mathematics?
   - What can be done (for example, training) by my class to meet community needs?
   - What skills and resources does the community have that could benefit my students?
   - What kind of relationships can my class forge with the community?

5. Establish support groups consisting of school personnel and community members. The goals of these groups are to:
   - Share teaching and learning experiences both in and out of school
   - Discuss research and theory related to learning
   - Act as mentors and coaches for one another
   - Connect goals of the community with goals of the school

Activities for Schools, Parents, and Community Members

The following are activities that groups such as your PTA, church, and local Chamber of Commerce can do together with your schools.

1. Visit your school informally for discussions using the checklist on page 23.

2. Consider ways that schools and community members can work together to provide:
   - Materials for a rich learning environment (e.g., real literature in print and audio form, computers)
   - Opportunities for students and teachers to learn out of school
   - Opportunities for students to access adults as role models, tutors, aides, and experts
   - Opportunities for students to provide community services such as surveys, newsletters, plays, and tutoring
   - Opportunities for students to participate in community affairs
   - Opportunities for administrators, teachers, or students to visit managers and company executives

3. Promote school and community forums to debate the national education goals:
   - Invite your local television and radio stations to host school and community forums.
   - Have "revolving school/community breakfasts" (community members visit schools for breakfast once or twice a month, changing the staff and community members each time).
   - Gather information on the national education goals and their assessment.
   - Gather information on alternative models of schooling.
   - Gather information on best practices and research in the classroom.
   - Examine the vision of mathematics teaching and learning set forth in the NCTM Standards.
   - Invite mathematics teachers in your area to discuss the changes being recommended by the NCTM Standards.

Some of the important questions and issues to discuss in your forums are:

- Have we reviewed the national education goals documents to arrive at a common understanding of each goal?
- What will students be like who learn in schools that achieve the goals?
- What must schools be like to achieve the goals?
- Do we agree with the goals, and how high do we rate each?
- What is the reason for the pessimism about their achievement?
- How are our schools doing now in terms of achieving each?
- Why is it important for us to achieve the goals?
- What are the consequences for our community if we don't achieve them?
- What assumptions are we making about the future in terms of Knowledge, Technology and Science, Humanities, Family, Change, Population, Minority Groups, Ecology, Jobs, Global Society, and Social Responsibility? Discuss in terms of each of the goal areas.

4. Consider ways to use *Children as Problem Solvers* to promote understanding and commitment from school staff, parents, and community members for mathematics understanding.

Checklist for Excellence in Mathematics Instruction

The items below are based on best practices of the teachers and researchers in Program 2. The checklist can be used to look at current practices in your school and to jointly set new goals with parents and community groups.

Vision of Learning

- Meaningful learning experiences for students and school staff
- High enjoyment of problem solving in mathematics
- Restructuring to promote learning in the classroom
- High expectations for learning for all students as set forth in the NCTM Standards
- A community of problem solvers in the classroom and in the school
- Teachers and administrators committed to achieving the National Education Goals as exemplified in the NCTM Standards

Curriculum and Instruction

- Curriculum that calls for a comprehensive repertoire of learning and problem-solving strategies
- Collaborative teaching and learning involving student-generated questioning and sustained dialogue among students and between students and teachers
- Teachers building new information on student strengths
- Authentic tasks in the classroom such as solving everyday problems, collecting and analyzing data, investigating patterns, and keeping journals
- Opportunities for students to engage in learning out of school with community members
- Homework that is challenging enough to be interesting but not so difficult to cause failure
- Respect for multiple cultures and perspectives
- Rich learning environment with places for children to solve math problems and think on their own
- Instruction that enables children to develop an understanding of math concepts

Assessment and Grouping

- Assessment that informs and is integral to instruction
- Performance-based assessment such as portfolios that include drafts, projects, and problem-solving logs
- Multiple opportunities to be involved in heterogeneous groupings, especially for students at risk
- Public displays of student work and rewards

Staff Development

- Opportunities for teachers to attend conferences and meetings for mathematics instruction
- Teachers as researchers, working on research projects
- Teacher or school partnerships/projects with colleges and universities
- Opportunities for teacher to observe and coach other teachers
- Opportunities for teachers to try new practices in a risk-free environment

Involvement of the Community

- Community members and parents' participation in mathematics instruction as experts, aids, guides, or tutors
- Active involvement of community members on task forces for curriculum, staff development, assessment, and other areas vital to learning
- Opportunities for teachers and other school staff to visit informally with community members to discuss the life of the school, resources, greater involvement of the community

Policies for Students at Risk

- Students at risk integrated into the social and academic life of the school
- Policies/practices to display respect for multiple cultures and role models
- Culture of fair assessment practices

SECTION 5

Biographical Information

Videoconference Participants

Thomas Carpenter is the Associate Director of the Wisconsin Center for Educational Research and a Professor of Curriculum and Instruction at the University of Wisconsin, Madison. For ten years he has researched how children learn mathematics, focusing on how math concepts are developed by primary age children. For the past six years, he has studied how to apply the resulting knowledge to classroom instruction and help teachers understand how children think about mathematics. He is the author of a chapter in the *Handbook on Teaching* which integrates research on teaching and learning. His research has been published in numerous national and international research journals. Currently, Dr. Carpenter serves as editor of the *Journal for Research in Mathematics Education*.

Susan Gehn is a 1st grade teacher at Cottage Grove Elementary in Cottage Grove, Wisconsin. Instead of using a textbook, Ms. Gehn prefers that her students use a problem-solving approach, based on Cognitively Guided Instruction (CGI) research, to investigate mathematics. She has shared her knowledge and experience by conducting inservice workshops and acts as a mentor teacher for teachers exploring the principles of Cognitively Guided Instruction. Ms. Gehn has participated at the American Educational Research Association (AERA) in a panel discussion on CGI and is a member of the National Council of Teachers of Mathematics (NCTM).

Beau Fly Jones is Director of Curriculum, Instruction, and Assessment at the North Central Regional Educational Laboratory. Dr. Jones holds a Ph.D. in Educational Psychology from Northwestern University. She has expertise in the areas of cognitive learning and effective comprehension instruction. Before becoming a member of the NCREL staff, Dr. Jones served as Special Projects Coordinator for the Chicago Public Schools, and was a developer for their instructional materials in several areas, particularly social studies and reading. Since her arrival at NCREL in 1985, she has published over 25 articles and books on dimensions of thinking, cognitive instruction, and applications of technology to curriculum development. She also has been responsible for a number of national videoconferences.

Judson Hixson is Director of Professional Preparation and Development at the North Central Regional Educational Laboratory (NCREL) and Associate Director of Technology and Information at the Midwest Regional Center for Drug-Free Schools and Communities (MRC). Mr. Hixson is a Ph.D. candidate at the University of Chicago and holds both a Bachelor and Master of Arts degree in Educational Psychology from the University of Chicago. Before joining NCREL, Mr. Hixson was Administrative Coordinator in the Chicago Public Schools. He also served as Educational Director for the Chicago Urban League, and as on-loan consultant to the National Urban League for educational policy development and congressional relations. He has teaching experience at all grade levels, as well as experience in university administration. He has also served as consultant to a wide variety of organizations, such as the U.S. Department of Justice Community Relations Division.

Jane King is a teacher in Boston's Back to School Program, an alternative middle school for older students with attendance problems. She became committed to the Boston Public Schools first as a parent, before changing careers to become a teacher. Mrs. King began involving herself in the Algebra Project when her son's school became active. She feels that the students are the most important resource for the Algebra Project, making it work through individual investigation. During the summer, Mrs. King works with the Island School Outbound program for middle school students from Boston.

Mary Lindquist is the Fuller E. Callaway Professor of Mathematics Education at Columbus College, Columbus, Georgia. She is currently the President-Elect of the National Council of Teachers of Mathematics (NCTM) and is serving on the Advisory Board of the National Center for Research in Mathematical Sciences Education and the Mathematical Sciences Education Board of the National Research Academy. Dr. Lindquist has experience in teaching mathematics and developing curriculum for all levels, kindergarten through college.

Jacqueline Rivers is the Boston Coordinator for the Algebra Project. As the Coordinator, she conducts ten-day teacher training sessions, provides teacher and administrative support, fosters the connection between middle schools and university mathematics professors, and encourages teachers to bring adults into the classroom. She is a consultant with the Algebra Centered Enrichment Program and continues to be dedicated to an after-school program at Freedom House. She was responsible for initially recruiting schools for the Boston area Algebra Project—identifying teachers, principals, and Directors of Curriculum and Instruction. Ms. Rivers is presently a member of the Azusa Christian Community which focuses on addressing the needs of inner-city residents.

Claude Snead is a senior at Cambridge Ridge and Latin High School in Boston, Massachusetts. He attributes his success in advanced mathematics to the Middle School Algebra Project. He believes the project gave him a strong math background and the opportunity to take advanced courses in preparation for college. This summer he was involved with the Freedom House program as an Algebra Intern. During the school year he is a member of the high school track team.

Dorothy Strong is the Director of the Bureau of Mathematics for the Chicago Public Schools. Dr. Strong serves as the main liaison between the Chicago Public Schools and the Algebra Project. She has written several textbooks and numerous curriculum publications, is a national consultant on mathematics education, and has worked in the Chicago Public Schools for 31 years. Dr. Strong has served on numerous national committees, including the Board of Directors of the National Council of Teachers of Mathematics. The President's National Advisory Council's Education Award is one of the numerous awards she has received. She served as President of the National Council of Supervisors of Mathematics and is currently President of the Benjamin Benneker Mathematics Association. Dr. Strong is committed to the mission of urban education.

SECTION 6

Important Resources in Mathematics

Cognitively Guided Instruction (CGI) is a model of curriculum development designed to increase learners' understanding by considering teachers, learners, and mathematics. The underlying assumption is that classroom instruction is determined by the decisions that teachers make which are directly influenced by teachers' knowledge and beliefs. The major tenets of CGI are that (1) instruction must be based on what each learner knows, (2) instruction should take into consideration how children's mathematical ideas develop naturally, and (3) children must be mentally active as they learn mathematics. All decisions about what and how to teach are dependent on what the child already knows and thinks. Studies have shown that CGI students solve more difficult mathematics problems and do as well or better on skills measures than non-CGI students. Tom Carpenter, National Center for Research in Mathematical Sciences Education, University of Wisconsin-Madison 53706.

NCTM Publications (A complete catalog of these and other resources is available from NCTM, 1906 Association Drive, Reston, VA 22091.)

Professional Standards for Teaching Mathematics. (1991, NCTM).

Curriculum and Evaluation Standards for School Mathematics. (1989, NCTM).

Algebra Teacher. A journal for K-8 teachers of mathematics. Special Issues of this journal include:

Number Sense (February, 1989)
Data Analysis (February, 1990)
Reaching All Students (February, 1991)

Yearbook: New Directions for Elementary School Mathematics. (1989, Paul R. Trafton, Ed.).

Mathematics for the Young Child. (1991, Joseph N. Payne, Ed.).

Algebra for Everyone. (1991, Edgar Edwards, Jr., Ed.).

Constructivist View on the Teaching of Learning of Mathematics, Monograph 4 in the Journal for Research in Mathematics Education Monograph Series (1990, Robert. B. Davis et al., Eds.).

Addenda Series, Grades K-6. (1991, Miriam Leiva, Ed.). Separate books for each grade will be available in this series that exemplify the NCTM Standards in the areas of data sense, spatial sense, number and operation sense, and patterns. These will also be available by topics across the grade span.

Addenda Series, Middle Grades. (1991, Fran Curcio, Ed.). This series will contain booklets on data sense, number sense, patterns and functions, geometry, and measurement.

University of Chicago School Mathematics Project (UCSMP) is a mathematics curriculum, published by Scott Foresman, designed to upgrade the experience of the average student. It consists of both curricular materials and teacher development strategies. For example, the elementary materials encourage teachers and students to explore math by investigating, informally but systematically, the basics of data gathering and analysis, probability, geometry, and algebra. The transition mathematics course consolidates the arithmetic of the previous grades and prepares the student for algebra and geometry. UCSMP views mathematics as a tool for life and emphasizes how teachers can integrate math with other subjects and use technology, such as calculators and computers, to help students develop mathematical understanding. Zalman Usiskin, Director, UCSMP, University of Chicago, Chicago IL 60637.

The Adventures of Jasper Woodbury is a research and development project by the Cognition and Technology Group at Vanderbilt University. The goal of this project is to create a shared learning environment that permits sustained exploration by students and teachers and enables them to understand the kinds of problems and opportunities that experts in various areas encounter and the knowledge that these experts use as tools. Each video in the Jasper series has a main story ending in a challenge to solve a problem. Students have to generate the relevant sub-problems using data embedded in the story. The series currently has four episodes which are designed on the following principles: a video-based presentation, a narrative format, generative learning, embedded data design, complex problems, adventures designed in pairs (to promote transfer), and links across the curriculum. The Cognition and Technology Group, Learning Technology Center, Peabody College of Vanderbilt University, Nashville, TN 37240.

SECTION 7

Glossary

Algorithm A prescribed process or set of steps which, when followed, always leads to the solution of the problem. For example, long division is an algorithm for dividing one number by another.

Assessment Techniques used to determine how well students are achieving, what students know and can do, and to what extent an instructional strategy or program is working.

Coaching Providing support in studying new skills, polishing old ones, and encouraging change.

Collaborative Groups A temporary grouping structure used primarily for developing attitude outcomes. Students of varying abilities work together to solve a problem or to complete a project.

Cooperative Learning Students working together in small heterogeneous groups to achieve a common goal.

Curriculum An operational plan for instruction that details what students are to know, how they are to achieve the curricular goals, what teachers are to do to help students, and the context in which learning and teaching occur.

Formal Mathematics Knowledge Knowledge, usually acquired in school, about how to use mathematical symbols and symbolic procedures to solve mathematical problems.

Informal Mathematics Knowledge Knowledge about mathematics that children learn through experience outside of instruction. For example, children outside of school invent procedures, count on their fingers, represent math problems with objects, and use derived facts before they have memorized number facts.

Inquiry A process of investigating a phenomena, devising and working through a plan, and proposing a solution to the problem. A disposition of inquiry is necessary for sucessful problem solving.

Manipulatives Physical objects used to represent, or model, a problem situation. For example, base ten blocks can be used to represent problems involving the multiplication of numbers.

Modeling Showing a student how to do a task with the expectation that the student will then emulate the model. Modeling often involves talking about how one thinks through a task.

Representation A model, or alternate form, of a problem situation or aspect of a problem situation used in finding a solution. For example, problems can be respresented by objects, pictures, words, or mathematical symbols.

Structured Language Language spoken by children as they solve mathematical problems that coincides with the symbolic language of math.

Symbol System A set of symbols and rules for manipulating them to represent and solve mathematical problems. For example, the collection of symbols including =, +, -, /, etc., and the rules for their combination and representation allow one to perform arithmetic operations.

Symbol A mark, such as an equal sign (=), that stands by convention for a particular concept.

Understanding The relationships between segments of knowledge that the learner forms through thinking, reasoning, and problem solving.

SECTION 8

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Silver, E.A. (Ed.) (1985). Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Erlbaum.

Product Order Form

Qty.	Price	Order #	Title/Author
____	$300	RES 091	**Restructuring to Promote Learning in America's Schools** (1990). A series of nine, 2-hour videotapes and guidebooks. Produced by NCREL and PBS. ■ The New Definition of Learning: The First Step for School Reform ■ The Thinking Curriculum ■ The Collaborative Classroom: Reconnecting Teachers and Learners ■ Multidimensional Assessment: Strategies for the Classroom ■ Schools and Learning Communities ■ Many Roads to Fundamental Reform: Getting Started ■ Many Roads to Fundamental Reform: Continuing the Journey ■ The Meaning of Professional Development in the 21st Century ■ Reconnecting Students at Risk to the Learning Process Individual videotape/guidebook sets are available for $35. Please specify which Videoconference you want.
____	$10	HST 703	**Dimensions of Thinking: A Framework for Curriculum and Instruction** (1988). Robert J. Marzano, Ronald S. Brandt, Carolyn Sue Hughes, Beau Fly Jones, Barbara Z. Presseisen, Stuart C. Rankin, and Charles Suhor.
____	$10	HST 705	**Strategic Teaching and Learning: Cognitive Instruction in the Content Areas** (1987). Beau Fly Jones, Annemarie Sullivan Palincsar, Donna S. Ogle, and Eileen Glynn Carr.
____	$6	TC 901	**Managing Instruction for Equity and Excellence** (Facilitator's Manual) (1989). Beau Fly Jones (Ed.).
____	Free	VTC 101	**Video Teleconferencing: A Potentially Powerful Vehicle for Staff Development** (1987). Article by Beau Fly Jones.
____	Free	ED 091	**EDTALK: What We Know About Mathematics Teaching and Learning** (1991). Nancy Kober. Distributed at no charge while supplies last.

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